11-4 Geometric Series

sum of a geometric sequence

$2 + 6 + 18 + 54 + 162$

$r = 3$

$S_5 = 2 + 6 + 18 + 54 + 162$

$-3S_5 = -6 - 18 - 54 - 162 - 486$

$-2S_5 = 2 - 486$

$S_5 = \frac{2-486}{-2} = \frac{a_1 - r \cdot a_n}{1-r}$

$= 242$

$a_n = a_1 r^{n-1}$

$S_n = \frac{a_1 - r a_n}{1-r}$

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$S_n = \frac{a_1 - a_1 r^n}{1-r}$ $r \neq 1$

$S_n = \frac{a_1(1-r^n)}{1-r}$

Ex 1:
Find the sum and then find n.
$a_1 = 7776$
$a_n = 6$
$r = \frac{-1}{6}$

$S_n = \frac{a_1 - r \cdot a_n}{1 - r}$

$= \frac{7776 - \left(-\frac{1}{6}\right)6}{1 - \frac{-1}{6}}$

$S_n = 6,6666$

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$\frac{6,6666}{1} = \frac{7776\left(1 - \left(-\frac{1}{6}\right)^n\right)}{1 + \frac{1}{6}}$

$7777 = 7776\left(1 - \left(-\frac{1}{6}\right)^n\right)$

Ex 2:

$a_1 = ?$
$S_8 = 765$

$r = 2$

$765 = \frac{a_1\left(1 - (2)^8\right)}{-1}$

$3 = a_1$

Sigma Notation

$\sum_{n=1}^{12} 3\,2^{n-1}$

$r = 2$
$a_1 = 3$
$a_{12} = 6144$

$S_{12} = \frac{3 - (2)(6144)}{1 - 2}$

$= 12285$

Put the following series into sigma notation.

$48 + 24 + 12 + 6 + \ldots + \frac{3}{8}$

$a_n = a_1 \cdot r^{n-1}$

$\frac{3}{8} = 48\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{128} = \left(\frac{1}{2}\right)^{n-1} \rightarrow \left(\frac{1}{2}\right)^7 = \left(\frac{1}{2}\right)^{n-1}$

$7 = n-1$

$8 = n$

$\sum_{n=1}^{8} 48\left(\frac{1}{2}\right)^{n-1}$

29. **HEALTH** Contagious diseases can spread very quickly. Suppose five people are ill during the first week of an epidemic and that each person who is ill spreads the disease to four people by the end of the next week. By the end of the tenth week of the epidemic, how many people have been affected by the illness?

$a_1 = 5$

$r = 4$

$a_{10} = 5(4)^9$

$S_{10} = \frac{5 - 4(a_n)}{1 - 4}$

$1,747,625$

HW
p597
15-27odd, 30, 33, 35, 39, 41, 43, 47